SG:

$$\hat{H}\psi_t = i\hbar \frac{d\psi_t}{dt} \quad , \quad \underline{AB}: \ \psi_{t_0}$$

$$\int_{\mathbb{R}} dx \, |\psi_{t_0}(x)|^2 = 1 \quad (*)$$

$$\Rightarrow \psi_t \, , \, t > t_0 \, , \quad \int_{\mathbb{R}} dx \, |\psi_t(x)|^2 = 1 \quad (*)$$

$\underline{\mathcal{P}}$: $\hat{p}$

EWP: $\hat{p} u_{p'} = p' u_{p'} \quad , \quad p' \in \mathbb{R}$

$$u_{p'}(x) = \frac{1}{\sqrt{2\pi\hbar}} e^{\frac{i}{\hbar} p' x}$$

$$\int_{\mathbb{R}} dx \, u^*_{p'}(x) u_{p''}(x) = \delta(p' - p'') \quad (*)$$

$$\langle f, g \rangle := \int_{\mathbb{R}} dx \, f^*(x) g(x)$$

$$\| f \| := \sqrt{\langle f, f \rangle}$$

$\langle \psi_{t_0}, \psi_{t_0} \rangle = 1$ oder $\| \psi_{t_0} \| = 1$

$\langle \psi_t, \psi_t \rangle = 1$ oder $\| \psi_t \| = 1$

$\langle u_{p'}, u_{p''} \rangle = \delta(p' - p'')$

$$\left.\begin{aligned} \psi_t(x) &= \int_{\mathbb{R}} dp'\, \tilde{\psi}_t(p')\, u_{p'}(x) \\ \text{mit} \quad \tilde{\psi}_t(p') &= \int_{\mathbb{R}} dx\, u_{p'}^*(x)\, \psi_t(x) \end{aligned}\right\} (*)$$

$$\psi_t(x) = \int_{\mathbb{R}} dp' \underbrace{\langle u_{p'}, \psi_t \rangle}_{\tilde{\psi}_t(p')} u_{p'}(x)$$

$$W(p',t) = |\tilde{\psi}_t(p')|^2 \qquad (*)$$

$$W(p',t) = |\langle u_{p'}, \psi_t \rangle|^2$$

$$\bar{p}(t) = \int_{\mathbb{R}} dp\, p\, W(p,t) = \int_{\mathbb{R}} dx\, \psi_t^*(x)\, (\hat{p}\psi_t)(x) \qquad (*)$$

$$\bar{p}(t) = \int_{\mathbb{R}} dp\, p\, W(p,t) = \langle \psi_t, \hat{p}\psi_t \rangle$$

Bsp: linearer harmonischer Oszillator

$$\hat{H} = H(\hat{x}, \hat{p}) = \frac{\hat{p}^2}{2m} + \frac{m\omega_0^2}{2}\hat{x}^2$$

EWP: $\hat{H} u_n = E_n u_n \quad , n \in \mathbb{N}_0$

$$E_n = (n + \tfrac{1}{2})\hbar\omega_0$$

$$u_n(x) = \sqrt{\frac{\alpha}{2^n n! \sqrt{\pi}}}\, e^{-\frac{(\alpha x)^2}{2}} H_n(\alpha x)$$

$$\alpha := \sqrt{\frac{m\omega_0}{\hbar}}$$

$$\int_{\mathbb{R}} dx\, u_{n'}^{*}(x)\, u_{n''}(x) = \delta_{n'n''} \qquad (*)$$

$$\left.\begin{aligned} \psi_t(x) &= \sum_{n=0}^{\infty} c_n(t)\, u_n(x) \\ \text{mit} \quad c_n(t) &= \int_{\mathbb{R}} dx\, u_n^{*}(x)\, \psi_t(x) \end{aligned}\right\} (*)$$

$$W_n(t) = |c_n(t)|^2 \qquad (*)$$

$$\bar{E}(t) = \sum_{n=0}^{\infty} E_n W_n(t) = \int_{\mathbb{R}} dx\, \psi_t^{*}(x)\, (\hat{H}\psi_t)(x) \qquad (*)$$

$$\langle u_{n'}, u_{n''} \rangle = \delta_{n'n''} \qquad *)$$

$$\psi_t(x) = \sum_{n=0}^{\infty} \underbrace{\langle u_n, \psi_t \rangle}_{c_n(t)} u_n(x) \qquad *)$$

$$W_n(t) = |\langle u_n, \psi_t \rangle|^2$$

$$\bar{E}(t) = \sum_{n=0}^{\infty} E_n W_n(t) = \langle \psi_t, \hat{H}\psi_t \rangle$$

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*) gewöhnliche Vektorrechnung im $\mathbb{R}^3$

$$\vec{e}_{n'} \cdot \vec{e}_{n''} = \delta_{n'n''}$$

$$\vec{c} = \sum_{n=1}^{3} \underbrace{(\vec{e}_n \cdot \vec{c})}_{c_n} \vec{e}_n$$